UNNS Operational Grammar:
Inletting, Inlaying, Trans–Sentifying, and Repair & Normalization

UNNS Research Notes

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Abstract

This document formalizes a unified operational grammar for the Unbounded Nested Number Sequence (UNNS) substrate. Four complementary primitives are described in detail: *Inletting* (injection of external data), *Inlaying* (internal embedding of motifs), *Trans–Sentifying* (mapping invariants to perceptual / semantic channels), and *Repair & Normalization* (DNA-inspired correction operators). For each primitive we provide definitions, examples, algorithmic recipes, stability lemmas or propositions, and a discussion of interactions. The goal is a durable, implementation-oriented treatment suitable for inclusion in the UNNS corpus or as a stand-alone specification.

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1 Introduction

UNNS (Unbounded Nested Number Sequences) is a recursive substrate for producing nested algebraic, geometric and topological structures from finite recurrence data. To make UNNS a practical discipline (for numerics, physics, visualization, and outreach) we define a small, precise grammar of operations that control how data enter the substrate, how the substrate modifies itself, how invariants are exported for perception, and how errors are corrected. The four primitives described here—*Inletting*, *Inlaying*, *Trans–Sentifying*, and *Repair & Normalization*—compose into robust pipelines for experimentation and deployment.

2 Preliminaries: the UNNS substrate

We summarize minimal notation and assumptions used below.

Definition 2.1 (UNNS substrate)**.** *A* UNNS substrate $\mathcal{U}$ *is an abstract data structure supporting:*

- *finite-order linear recurrences (sequences)* $u_{n+r} = \sum_{j=1}^{r} c_j u_{n+r-j}$*,*
- *mesh-labeled data for discrete exterior calculus (edge labels* $c(e)$*, face residues, etc.),*
- *diagnostics (echo residues, growth factors, UPI — the UNNS Paradox Index).*

We assume $\mathcal{U}$ *admits local models (finite windows) where linear-algebraic techniques (companion matrices, spectral radius) are meaningful.*

Notation: C denotes a companion matrix associated to a recurrence, $\rho(C)$ its spectral radius. We write $\mathrm{UPI}(\mathcal{R})$ for the paradox index measured on a region $\mathcal{R}$.

3 UNNS Inletting

3.1 Definition and intent

Definition 3.1 (UNNS Inletting)**.** *A* UNNS inletting *is a morphism*

$$\iota : D \longrightarrow \mathcal{U},$$

where D is finite external data (numeric samples, boundary assignments, measured coefficients), such that $\iota(D)$ is placed into $\mathcal{U}$ in a manner compatible with its recurrence rules, with either (a) no violation of stability thresholds, or (b) followed immediately by repair/normalization.

Intuitively, inletting is the canonical way to couple real-world data or experiment inputs to the UNNS substrate: initial seeds, boundary edge values, measured cochains, or source terms.

3.2 Examples

Example 3.2 (Sequence seeding)**.** *Given an order-r UNNS sequence, an inletting may overwrite the initial seed vector $(u_0, \dots, u_{r-1})$ with data from D (e.g. $u_0 = 2$, $u_1 = 5$). The recurrence subsequently propagates the new trajectory.*

Example 3.3 (Boundary coefficient inletting in a mesh)**.** *For a DEC-labeled simplicial mesh, define $\iota(D)$ by assigning measured edge weights on the boundary. Solving interior cochains uses these boundary inlets.*

3.3 Stability lemma

Lemma 3.4 (Bounded inletting)**.** *Let $\mathcal{U}$ be a linear recurrence substrate with companion matrix C satisfying $\rho(C) < 1$. If an inletting $\iota : D \to \mathcal{U}$ injects bounded data $|d| \leq M$, then the produced UNNS trajectory is uniformly bounded by $O(M/(1 - \rho(C)))$ (constants depend on r).*

Sketch. A linear stable recurrence with spectral radius $\rho(C) < 1$ acts as a contraction on the space of sufficiently regular seed vectors. Responses to bounded inputs decay geometrically; standard linear system bounds yield the stated estimate. □

3.4 Algorithmic recipe (Inletting)

1. **Extract** D (samples, boundary values, coefficients).
2. **Check compatibility:** verify order, data types, and ring constraints (optionally project).
3. **Apply ι:** write values into $\mathcal{U}$ at designated locations.
4. **Measure diagnostics:** compute local residues, $\rho(C)$ proxies, UPI in affected zones.
5. **If thresholds exceeded,** call Repair & Normalization routine (Section 6).

4 UNNS Inlaying

4.1 Definition

Definition 4.1 (UNNS Inlaying)**.** *A* UNNS inlaying *is an internal operator*

$$\mathcal{I}_{P \to L} : \mathcal{U} \to \mathcal{U},$$

which selects an existing motif P within $\mathcal{U}$ (a subsequence, local stencil, or subgraph), transforms it (scaling, phase, projection), and inserts the transformed motif $\widetilde{P}$ at a target location L so that the recursive dynamics of $\mathcal{U}$ extend naturally over $\widetilde{P}$.

4.2 Examples

Example 4.2 (Sequence motif inlaying)**.** *Select a subsequence* $P = (u_k, \ldots, u_{k+m-1})$*, produce a phase-shifted copy* $\widetilde{P}$*, and write it into indices* $j, \ldots, j + m - 1$ *as seeds or local coefficients.*

Example 4.3 (Mesh motif inlaying)**.** *Copy a triangular patch of edge coefficients* P *and paste a projected copy into a distant patch* L*, producing controlled change in local echo residues.*

4.3 Local stabilization lemma

Lemma 4.4 (Local stabilization by contractive inlay)**.** *Suppose region* R *of* $\mathcal{U}$ *has an effective local companion with spectral radius* $\rho_R > 1$*. If there exists a motif* P *whose companion* C_P *satisfies* $\rho(C_P) < 1$ *and* $\mathcal{I}_{P\to L}$ *replaces the local update in* R *by* C_P *(with small boundary coupling), then* R *becomes locally contractive: local growth is controlled by* $\rho(C_P)$*.*

Sketch. Replacing local update operator changes the local monodromy matrix. If boundary coupling is small (norm small), the interior spectrum is dominated by $\rho(C_P)$; therefore modes decay at the rate governed by $\rho(C_P)$. □

4.4 Algorithmic recipe (Inlaying)

1. **Select motif** P from stable area; compute its companion C_P and $\rho(C_P)$.
2. **Transform** as needed (scale, phase shift, ring projection).
3. **Simulate insertion** in a local model around L; measure predicted residues and local spectral radius.
4. **Accept/reject** based on thresholds; commit insertion if acceptable.
5. **Monitor** and, if necessary, apply repair operators.

5 UNNS Trans–Sentifying

5.1 Definition

Definition 5.1 (UNNS Trans–Sentifying)**.** *A* Trans–Sentifying protocol $\mathcal{T}$ *is a mapping*

$$\mathcal{T} : \mathcal{U} \longrightarrow \mathcal{S},$$

where $\mathcal{S}$ *is a perceptual or semantic space (audio, visual, symbolic, or machine embeddings).* $\mathcal{T}$ *consists of an* encoding layer *and a* rendering layer*:*

$$\mathcal{U} \xrightarrow{\text{encode}} \mathbf{f} \in \mathbb{R}^k \xrightarrow{\text{render}} \textit{channel output.}$$

5.2 Design constraints

- **Fidelity:** preserve the most salient UNNS invariants (dominant eigenvalues, echo spectra).
- **Stability:** avoid amplification of numerical/structural noise when mapping to perceptual signals.
- **Invertibility (optional):** allow approximate back-mapping for diagnostics or control.
- **Accessibility:** choose renderings that are informative and interpretable by intended users.

5.3 Canonical encoding recipes

Given a local region $\mathcal{R} \subset \mathcal{U}$, produce feature vector $\mathbf{f}(\mathcal{R})$:

$$\mathbf{f} = \big(\rho(C_{\mathcal{R}}),\ \max|e|,\ \text{avg phase}(c_j),\ \text{GaussResidues}, \dots\big).$$

Mapping examples:

- **Audio:** map frequencies $f_i = f_0 \cdot \exp(\alpha \arg(c_i))$, map residues to amplitude envelopes.
- **Visual:** map residue magnitude to color luminance, map phase to hue, draw nested lattices as animated tilings.
- **Semantic:** quantize Gauss/Jacobi sum residues into tokens; feed to embedding model for clustering.

5.4 Properties and safety

Proposition 5.2 (Noise-tolerant mapping)**.** *If encoding uses normalized features (e.g. residues normalized by local max) and rendering applies smoothing, then small perturbations in $\mathcal{U}$ cause bounded perceptual changes (Lipschitz property).*

Sketch. Normalization and smoothing act as low-pass filters; the composed map is Lipschitz with constant bounded by the normalization and smoothing parameters. □

6 Repair & Normalization (DNA-inspired)

6.1 Overview

Repair and normalization are corrective primitives that ensure stability and meaningful transsentified outputs. They are inspired by biological mechanisms:

- *Proofreading* — local smoothing toward predicted value.
- *Excision + Refit* — remove a corrupted local patch and recompute via constrained fit.
- *Mismatch Projection* — project coefficients/terms to admissible algebraic rings.
- *Homologous Replacement* — paste template motif from stable area.
- *Global Renormalization* — rescale whole substrate or region to control amplitude/growth.
- *Gauge Adjustment* — apply discrete gauge transforms to reduce cochain residues.

6.2 Formal operators

We denote repair operators by $\mathcal{R}$ with subscripts indicating type.

Proofreading $\mathcal{R}_{pf}(i, \eta)$ On index i, with strength $\eta \in (0, 1]$:

$$u_i \leftarrow (1-\eta)\, u_i + \eta\, \widetilde{u}_i,$$

where $\widetilde{u}_i$ is the recurrence prediction from neighboring seeds.

Excision + Refit $\mathcal{R}_{ex}(I)$ For contiguous index set $I = [a, b]$:

1. Remove u_j for $j \in I$.
2. Fit coefficients locally (least-squares + regularization) on a surrounding window.
3. Reconstruct u_j for $j \in I$ using fitted recurrence.

Mismatch Projection $\mathcal{R}_{pr}$ Project scalar/complex coefficients to nearest element in ring (e.g. Gaussian integers):

$$c \mapsto \operatorname{argmin}_{r \in \mathbb{Z}[i]} |c - r|.$$

Homologous Replacement $\mathcal{R}_{hr}(P \to L)$ Replace region L by template motif P after alignment (scale/phase).

Global Renormalization $\mathcal{R}_{gn}(\lambda)$ Scale values to control global norm:

$$u_n \leftarrow \lambda\, u_n, \qquad \lambda = \frac{1}{\sqrt{\frac{1}{N}\sum_{n=1}^{N} |u_n|^2}}.$$

6.3 Mathematical properties

Proposition 6.1 (Local monotonicity of proofreading)**.** *Assume the recurrence predictor is linear and denote local residue* $r_i = \widetilde{u}_i - u_i$*. After* $\mathcal{R}_{pf}(i, \eta)$ *the local residue becomes* $(1 - \eta)r_i$*. Thus* $|r_i|$ *strictly decreases for* $\eta \in (0, 1]$*.*

Proof. By substitution:

$$\widetilde{u}_i - u_i^{\text{new}} = \widetilde{u}_i - (1 - \eta)u_i - \eta\widetilde{u}_i = (1 - \eta)(\widetilde{u}_i - u_i) = (1 - \eta)r_i.$$

□

Remark 6.2. *Other operators (excision, homologous replacement) can be shown to reduce appropriate global diagnostics (energy, max residue) under mild regularization assumptions; details depend on problem-specific norms.*

6.4 Repair policy: where and when to apply repairs

A practical repair policy:

1. Monitor diagnostics: local residues, growth factors, $\rho(C)$, UPI.
2. If a single index exceeds a small threshold τ_1, apply $\mathcal{R}_{pf}(i, \eta)$.
3. If a contiguous block exceeds larger threshold τ_2, apply $\mathcal{R}_{ex}$ or $\mathcal{R}_{hr}$.
4. If coefficients violate ring constraints, apply $\mathcal{R}_{pr}$.
5. Periodically apply $\mathcal{R}_{gn}$ to keep amplitudes tractable for visualization / trans-sentifying.

7 Interplay: a unified pipeline

The four primitives form a cyclic pipeline:

The directionality above is schematic: in practice the operations interleave—e.g. repairs are applied immediately after inletting, trans-sentifying is used for diagnostics as well as presentation, and inlaying may itself be triggered by trans-sentified feedback.

8 Examples and Worked Scenarios

8.1 Sine inletting + repair

1. Inlet a sampled sine into seeds of order r, fit recurrence via regularized least squares.
2. Compute $\rho(C)$. If $\rho(C) > 1.05$, increase regularization λ and refit (repair step).
3. Propagate and trans-sentify: show continuation and map residues to color.

8.2 Fibonacci motif inlaying for stabilization

1. Identify unstable region R with growth; select Fibonacci-like motif P from stable area.
2. Inlay P at L after projecting to admissible ring; simulate local evolution; if spectral reduction occurs, commit.

9 Implementation notes and experiment plan

- **Diagnostics module:** compute residues, growth factors, UPI, and spectral radii for local windows.
- **Repair module:** implement $\mathcal{R}_{pf}, \mathcal{R}_{ex}, \mathcal{R}_{pr}, \mathcal{R}_{hr}, \mathcal{R}_{gn}$ as callable operators.
- **Trans–Sentifying module:** pipeline for encoding $\mathbf{f}$ and rendering channels (audio via WebAudio or MIDI, visual via canvas/svg, semantic via embeddings).
- **Experiment plan:** synthetic corruption tests, adversarial corruptions, mesh FEEC tests, and human-subject perception studies for trans-sentifying mappings.

10 Concluding remarks

The four primitives—Inletting, Inlaying, Trans–Sentifying, and Repair & Normalization—form a compact, operational grammar for UNNS. They are intentionally modular and composable: inletting brings the world in, inlaying sculpts the substrate from within, trans-sentifying translates invariants to experience, and repair keeps the whole system resilient. This grammar supports rigorous analysis (spectral stability, projection to algebraic rings) while enabling applications across numerical PDEs, topology-inspired media, and pedagogical visualizations.